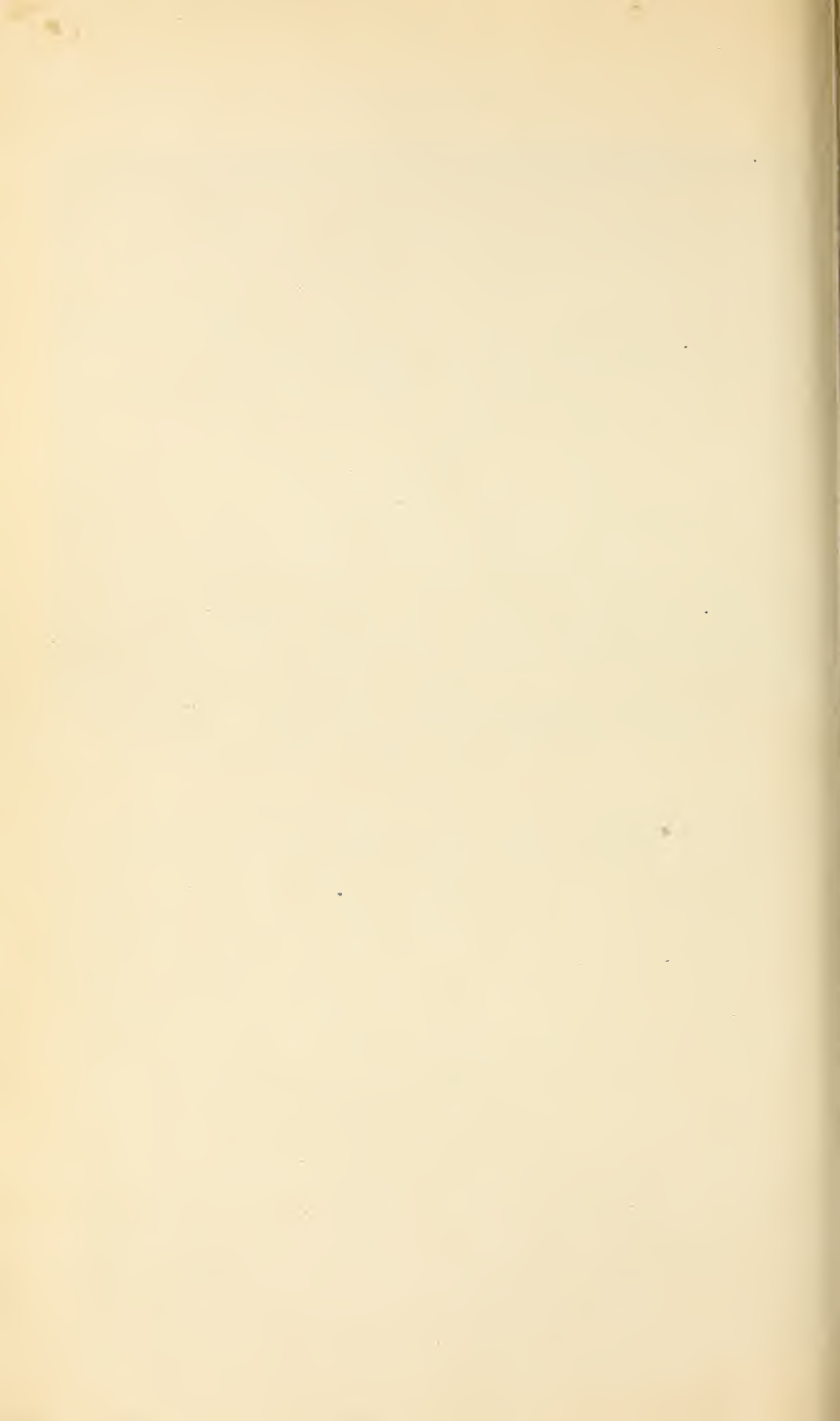






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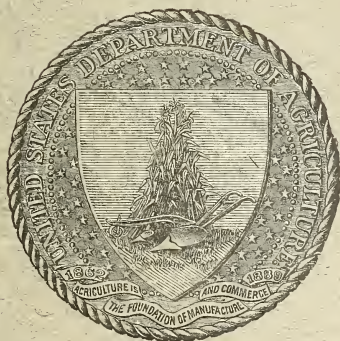
U. S. DEPARTMENT OF AGRICULTURE.

DIVISION OF SOILS.

# THE MECHANICS OF SOIL MOISTURE.

BY

LYMAN J. BRIGGS,  
PHYSICIST, DIVISION OF SOILS.



WASHINGTON:  
GOVERNMENT PRINTING OFFICE.

1897.



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## LETTER OF TRANSMITTAL.

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U. S. DEPARTMENT OF AGRICULTURE,  
DIVISION OF SOILS,  
*Washington, D. C., September 1, 1897.*

SIR: I have the honor to transmit herewith a paper upon the Mechanics of Soil Moisture, prepared by Mr. Lyman J. Briggs, physicist of this Division. The subject is necessarily treated in a technical way in order that it may be clearly understood by the student of agricultural science. It explains, however, more fully and clearly than ever before the actual cause of the capillary movement of water in soils and gives a much clearer knowledge of the laws and principles governing that movement than we have ever possessed. This is a subject of vast practical importance to the agriculturist, for the relation of his soils to water largely determines the class of crops which can be successfully grown upon them.

This paper is a valuable contribution to science, and I recommend that it be published as Bulletin No. 10 of this Division.

Respectfully,

MILTON WHITNEY,  
*Chief of Division.*

Hon. JAMES WILSON,  
*Secretary of Agriculture.*



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# THE MECHANICS OF SOIL MOISTURE.

## INTRODUCTION.

It is intended in this bulletin to present the application of certain dynamical principles to the problems attending the movement and retention of soil moisture. Among these problems may be mentioned the capacity of a soil for water, the adjustment of water between a dry and a wet soil, the relation of texture, structure, and temperature to the water capacity, and the effect of fertilizers upon the water content of a soil.

The extreme complexity of the texture and structure of soils renders very difficult any rigorous analysis of the phenomena connected with the movement of soil moisture. There are, however, two conceptions of the soil which have been productive of good results. One is to consider the soil as made up of very fine particles without regard to the form of the particles or capillary spaces, the only condition being that the interstitial spaces are so small that the amount of interstitial space represented on any small section taken in any direction through the soil should be practically constant. Such a structure has been assumed by Dr. Katao<sup>1</sup> as the basis of an extensive memoir on the movement of water in soils. In the other conception of the soil the particles are assumed to be of some simple geometrical form, such as the sphere. By means of different arrangements of these spherical particles, structures having different amounts of interstitial space may be obtained. A calculation of the interstitial space for several arrangements has been given by Soyka,<sup>2</sup> and a comparison with the structure of typical soils was made later by Professor Whitney.<sup>3</sup>

While the principles that will be developed here are independent of the structure of the soil, the various arrangements of the soil grains which are set forth in the last form of structure referred to will be used to illustrate the principles and forces involved.

<sup>1</sup>Ueber die Wasserbewegung in Boden. Bul. of College of Agriculture, Imperial University of Japan, vol. 3, No. 1, 1897.

<sup>2</sup>Forschungen auf dem Gebiete der Agrikultur-physik, B. 8, S. 1, 1885.

<sup>3</sup>Agricultural Science, vol. 3, p. 199, 1889.

## PROPERTIES OF WATER AFFECTING ITS RETENTION AND MOVEMENT IN THE SOIL.

The water contained in a soil may be considered to be of three kinds—gravitation water, capillary water, and hygroscopic water. Gravitation water is that portion which is in excess of the amount which the soil is able to retain under existing conditions, and is consequently free to drain away. The capillary water is that part which would be retained in the capillary spaces under these conditions, and which is capable of movement through capillary action. The hygroscopic water is that found on the surface of the grains, which is not capable of movement through the action of gravity or capillary forces.

The maximum amount of water which a given soil may contain depends upon the resultant effect of the two forces—gravitation and surface tension. The force due to gravity is proportional to the mass of the liquid considered, and is always directed vertically downward. In other words, it is the weight of the liquid. This mass of liquid would therefore leave the soil if not opposed by the action of some other force, the vertical component of which acting along the same line as the force of gravity must be equal to it and opposite in direction. The effective part of this force and the manner of its application, which is of the greatest importance in determining the movement of water in a soil, will be considered later.

### GRAVITATION OF WATER.

When a column of soil is saturated with water, its lower end being left in such a manner that water can escape, a gradual draining of the soil takes place. The rate of flow of this water gradually becomes less and less until finally it ceases. The amount of water which thus leaves the soil under the action of the force of gravity would be gravitation water, while that remaining in the capillary spaces of the soil would be capillary water.

There is no sharply-drawn line between these two quantities of water. The relative proportion depends, among other factors, upon the texture and structure of the soil, the surface tension of the soil water, the temperature, and the length of the column of soil considered. The importance of this last factor can be shown from the following considerations: Suppose we have 100 cubic inches of soil packed into 100 cubical boxes without bottom or top, each containing 1 cubic inch. Suppose the soil in each box to be saturated with water. There will be a free water surface at the top and at the bottom of each box. By means of forces existing in these surfaces the water in each cube is enabled to overcome the attraction of gravity, so that each cube is able to retain an amount of water equal to that necessary to produce saturation. In this case, therefore, there is no gravitation water. Suppose now that these cubical boxes are built up in a vertical column 100 inches high. The water surfaces previously existing at the top and bottom, respec-

tively, of two cubes now disappear when one cube is placed on top of the other. Instead of having 200 surfaces as before, we now have only two surfaces, and they are called upon to support a column of water one hundred times as high as before. This they are unable to do, and water begins to drip from the lower surface. This water, which was previously what we have termed capillary, now becomes gravitational in its nature, due simply to a change in the length of the column. If the water in the soil was held in vertical capillary tubes running throughout the length of the column the water in each tube would simply fall until the two surfaces were able to support the weight of the liquid. In the soil, however, we have a different condition. As the water begins to leave the upper part of the column new surfaces are developed *within* the soil. As the water continues to drain away, these surfaces become more efficient in a way which will be explained later, and finally there comes a time when the opposing force exerted by these surfaces is sufficient to balance the weight of the liquid and the drainage ceases.

As an example of the displacement of water toward the bottom of a vertical column of soil, it is of interest to consider some experiments by Professor King,<sup>1</sup> who showed that for coarse sands this displacement is very marked. A vertical tube, 42 inches in length, filled with sand, continued to discharge water at its lower end for forty days after saturation. At the end of that period the top 6 inches of soil contained but 2 per cent of water, while the bottom 6 inches averaged 18 per cent. These results were in a general way corroborated by actual field observations on an area protected from precipitation and evaporation. No experiments were made on soils of a finer texture. It will be shown later that in such soils the movement would probably be much less marked.

#### SURFACE TENSION.

It has been pointed out that the force exerted on a liquid through gravity varies only with the mass of the liquid. This force can therefore change in value only when the mass of the liquid varies. From this it follows that any movement of water which takes place after equilibrium has been once established must be brought about through a change in the amount of water present in the soil or through a change in the force opposing gravitation. It is therefore of importance to consider the nature of this opposing force which we call surface tension.

The phenomenon of surface tension is due to the existence of molecular forces. In a suspended drop of water, for example, the particles in the interior of the liquid are attracted equally in all directions by the other particles of the liquid. The resultant attraction on any particle in the interior is therefore zero, and it is free to move through the liquid. A particle on the surface of the drop, on the contrary, is not attracted equally on all sides, since the molecules of the gas surround-

<sup>1</sup>Fluctuations in the Level and Rate of Movement of Ground Water. F. H. King, U. S. Dept. of Agriculture Bul. No. 5, Weather Bureau, p. 25.



ing the drop exert less attraction upon the particle than is exerted by the particles of the liquid. The resultant attraction is therefore inward, along a line perpendicular to the surface of the liquid at that point. Now, the equations representing the behavior of the drop under the action of these forces are identical with those obtained if we imagine the drop inclosed in a water-tight membrane having a uniform tension. The action of the drop is therefore the same as if this imaginary membrane actually existed, and what we call surface tension is the tension that this ideal membrane would have to possess in order to produce the observed phenomena. This ideal membrane differs from all material membranes in that its tension does not change when the surface is increased. When the surface is extended, particles which were formerly in the interior are brought to the surface, so that the number of particles per unit of area of the surface always remains the same. The surface tension is also practically independent of the form of the surface. The mathematical theory indicates a very slight increase where the mean curvature is concave, and a slight decrease where it is convex. This difference is too small to be verified experimentally, but is of interest in connection with the fact that evaporation will take place from the convex surface at the same time that condensation is taking place on the concave surface. This in itself, then, must furnish a means of gradual adjustment of the water in the capillary spaces of a soil.

It is of importance here to distinguish clearly between surface tension and the effective force of a film. It has just been stated that the surface tension or the energy per unit area in the film is independent of the form and extent of the surface. The effective force or the pressure of the film, on the other hand, is dependent upon both the form and extent of the surface as well as upon the surface tension. This subject will be discussed more fully later.

When a drop of a liquid is placed upon the horizontal surface of a solid, it either rapidly spreads out upon the surface in a thin film, as a drop of water on a clean glass plate, or else remains in the form of a drop, with as little surface as possible in contact with the solid, as in the case of mercury on glass. If we know the surface tensions of the three surfaces which separate the solid and liquid, solid and gas, and liquid and gas, respectively, the action of the drop can be anticipated. This arises from the following considerations: Let the solid, liquid, and gaseous media in contact be represented by  $s$ ,  $l$ , and  $g$ , respectively. The surfaces separating these three media will meet in a line. Through any point in this line pass a plane perpendicular to the line, and let this section be represented by the plane of the paper in fig. 1.

At the point  $O$  there exist three forces equal in magnitude to the tensions of the surfaces of separation and directed along lines tangent to the surfaces at that point. In order that the system may be in equilibrium it is necessary that each of these forces shall exactly balance the resultant force of the other two. Let the vectors in the

figure represent the direction and magnitude of these three forces. If these forces are in equilibrium, then lines drawn parallel to the vectors and equal to them in length will form the three sides of a triangle. The exterior angles of this triangle represent the angles between the surfaces of separation of the three substances.

If a system is not in equilibrium, an adjustment must take place through change in the *direction* of the vectors, since their magnitude remains constant. Consequently, the surfaces tend to change until the necessary angle is obtained. In general, if the tension of the solid-liquid surface is greater than the sum of the tensions of the other surfaces, the liquid will gather itself up in a drop, as in the case of

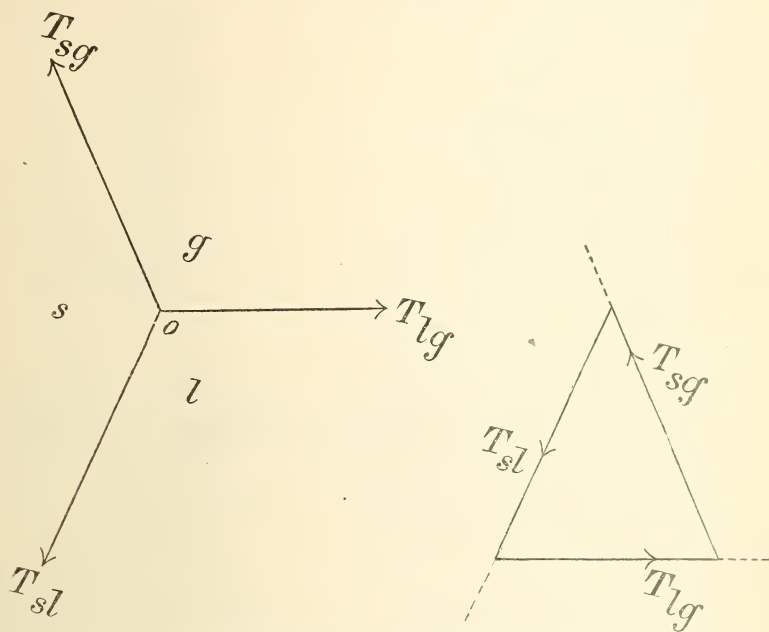


FIG. 1.—Condition of equilibrium among the surface tensions of three media in contact.

mercury. If, however, the solid-gas surface has a tension greater than the resultant of the other two, the liquid tends to spread out over the surface of the solid. If the tension of the liquid-gas surface is greater than the difference between the tensions of the liquid-solid and the gas-solid, then the liquid finally reaches a condition of equilibrium. The angle between the liquid-solid and the liquid-gas surfaces is known as the capillary angle.

In the case of oil on water the tension of the water-air surface is greater than the sum of the other two tensions, so that no triangle of forces can be formed. The system is therefore unstable and the oil spreads out on the water indefinitely in a film until it ceases to have the same physical properties as the liquid. Lord Rayleigh has made

use of the minimum thickness of this film as the basis of an estimation of the maximum diameter of oil molecules.

With the exception of mercury, water possesses a higher surface tension than any other substance which is liquid at ordinary temperatures. The surface tension of water<sup>1</sup> expressed in dynes per centimeter is 75.6 at 0° C. and 72.1 at 25°. The temperature coefficient is thus about -0.14 dynes per degree Centigrade. The surface tension of most aqueous solutions of salt is higher than that of water, and the surface tension increases with the concentration of the solution, as is shown in the following table:

*Surface tension of solutions of salts in water.*

Salt in solution.	Density.	Concentration. <sup>a</sup>	Temperature.	Surface tension.
			° C.	<i>Dynes per cm.</i>
KCl.....	1.170	25	15-16	82.8
KCl.....	1.101	15	15-16	80.1
KCl.....	1.046	7	15-16	78.2
NaCl.....	1.193	25	20	85.8
NaCl.....	1.107	15	20	80.5
NaCl.....	1.036	5	20	77.6
K <sub>2</sub> CO <sub>3</sub> .....	1.357	35	15-16	90.9
K <sub>2</sub> CO <sub>3</sub> .....	1.157	16	15-16	81.8
K <sub>2</sub> CO <sub>3</sub> .....	1.040	5	15-16	77.5
KNO <sub>3</sub> .....	1.126	19	14	78.9
KNO <sub>3</sub> .....	1.047	7	14	77.6
MgSO <sub>4</sub> .....	1.274	24	15-16	83.2
MgSO <sub>4</sub> .....	1.068	6	15-16	77.8

<sup>a</sup> Approximate weight of the dissolved substance in 100 parts by weight of the solution.

Most organic substances found in soils, especially those of an oily nature, being insoluble in water and hence most evident on the surface, lower the surface tension to a marked degree. The tension of soil extracts, therefore, is generally much lower than that of pure water, in spite of the presence of dissolved salts.

#### VISCOSITY.

It has been pointed out that the two great factors in determining the movement and retention of soil moisture are gravitation and surface tension. We have now to consider a modifying influence which is exerted upon these factors through the viscosity, or internal friction, of the liquid upon which these forces are acting, the effect of which is to retard the establishment of equilibrium. The relative viscosity of fluids may be determined by their rate of flow through capillary tubes under uniform conditions. Viscosity is generally expressed in terms of the coefficient of viscosity, which is numerically equal to the force necessary to maintain a flow of a layer of unit area past another layer of unit area with unit relative velocity. This coefficient is influenced by temperature to a considerable extent. If we take the viscosity<sup>2</sup> of water at 0° C.

<sup>1</sup> Smithsonian Physical Tables, 1896, p. 128. These values are a mean of the results obtained by Lord Rayleigh from the wave length of ripples (Phil. Mag., 1890), and by Hall from the direct measurement of the tension of a flat film (Phil. Mag., 1893).

<sup>2</sup> Smithsonian Physical Tables, 1896, p. 136.

to be 100, the viscosity at  $25^{\circ}$  C. is 50, at  $30^{\circ}$  is 45, and at  $50^{\circ}$  about 31. This great variation in viscosity with change of temperature is illustrated in the flow of water through soils, which King<sup>1</sup> found in his leaching experiments but failed to explain. He observed the rate of flow at  $9^{\circ}$  C. to be 6.15 grams per minute, while the rate of flow at  $32.5^{\circ}$  C. was 10.54 grams per minute. The ratio of the two rates of flow is 1.71. Now the viscosity of water at  $9^{\circ}$  C. as compared with water at  $0^{\circ}$  C. is 75.6, and  $32.5^{\circ}$  is 42.5. The ratio of the two viscosities is 1.77, which agrees very well with the ratio of the observed rates of flow.

The viscosity of gases in opposition to that of fluids increases with increase of temperature. Air, which is largely used in making so-called "permeability" determinations of soils, has a viscosity of 0.00017 ( $1+.00273 t$ ). An increase in temperature of  $40^{\circ}$  C. would therefore cause the coefficient of viscosity of air to increase one-tenth of its amount. This evidently should always be taken into consideration in determining the physical character of a soil. G. Ammon,<sup>2</sup> in using air to determine the relative permeability of soils, and neglecting the change in viscosity with temperature, found that the permeability of a soil decreased with increase of temperature. The rate of flow of air at the higher temperatures as observed by him, when corrected for viscosity, agrees with the flow observed for the lowest temperatures within the errors of experiment.

#### HYGROSCOPIC STATE.

Most solid substances when exposed to ordinary atmospheric conditions condense upon their surfaces a slight amount of moisture. This moisture adheres with remarkable tenacity, and can be completely driven off only by prolonged heating at temperatures above the boiling point of water. In some soils the presence of hygroscopic moisture is very marked, on account of the large amount of surface presented by the soil grains. Air-dried samples, in which all visible evidences of moisture have disappeared, still contain under ordinary atmospheric conditions moisture in the hygroscopic form, amounting in some soils to 8 to 10 per cent of the dry weight.

The table following, taken from a paper by Loughridge,<sup>3</sup> illustrates the variation of hygroscopic moisture in soils of different texture. These values were obtained by exposing the soil in a very thin layer to a saturated atmosphere, kept at a constant temperature, for a period of twenty-four hours.

<sup>1</sup> U. S. Weather Bureau Bul. No. 5, 1892, p. 66.

<sup>2</sup> Forschungen auf dem Gebiete der Agrikultur-Physik, B. 3, S. 209.

<sup>3</sup> Investigations in Soil Physics.—R. H. Loughridge. Report of the California Experiment Station, 1892-93, p. 70.



*Hygroscopic moisture of soils.*

Name and character.	Hygroscopic moisture.	Mechanical analysis.		Chemical analysis.	
		Clay.	Clay to 0.25 mm.	Soluble silicates.	Ferric hydrate.
Clay:		<i>Per cent.</i>	<i>Per cent.</i>	<i>Per cent.</i>	<i>Per cent.</i>
Black adobe .....	14.5	32.6	74.0	23.3	7.7
Red soil .....	14.2	24.8	57.1	54.0	9.5
Red mountain soil .....	13.7	52.2	67.9	26.9	29.7
Red volcanic soil .....	11.1	29.8	61.2	34.3	12.0
Alluvial soil .....	10.3	31.5	76.8	20.7	9.1
Alkali soil .....	2.6	26.1	54.0	.....	.....
Loams:					
Sediment soil .....	9.2	12.1	55.6	25.1	7.3
Granitic soil .....	5.9	11.9	32.8	21.0	6.2
Plains soil .....	4.9	10.5	34.9	16.5	6.6
Sandy:					
Gla bottom soils ....	3.5	3.2	8.7	8.9	7.4
Sandy soil .....	1.2	2.6	8.9	.....	.....
Do .....	.8	2.8	5.2	.....	.....

Under the conditions employed in the determinations given in the table it is not improbable that water was condensed in some of the more minute capillary spaces. Lord Kelvin<sup>1</sup> has shown that such a minute capillary surface is capable of condensing moisture even when evaporation is taking place from a neighboring plane surface of water. Some capillary spaces might therefore be able to hold minute quantities of water under conditions which would remove the water from larger spaces. In this way we might have some water held in a soil by capillary action, under conditions which would seem to indicate that the water content must be purely hygroscopic in its nature.

The nature of this thin film which constitutes the hygroscopic moisture is not definitely known. It may extend uniformly over the surface of the grains independently of their form or nature; or it may be discontinuous, occurring only in spots on the surface and depending to some extent on the form and nature of the grain. It would seem justifiable to assume that the amount of moisture thus held is proportional to the surface of the grains, but this conclusion is supported only in a very general way by the results given in the investigation quoted, from which the above table was compiled. Loughridge remarks, in explanation of this, that the clays may be considered as very complex substances made up not only of particles in a very fine state of division, but combined with ferric, aluminic, silicic, and humic hydrates existing in the soil in greatly varied amounts. Even if the presence of these hydrates did not directly influence the hygroscopic water of a soil their decomposition at the high temperature which it is necessary to maintain in order to drive off the hygroscopic moisture would introduce a disturbing factor in the value of the hygroscopic water content. On the other hand, it must be remembered that a mechanical analysis of a soil gives only in a very general way an idea of the surface area of the grains in a soil, and that a soil exposed to a saturated atmosphere is apt to acquire considerable moisture which is not strictly hygroscopic. The

<sup>1</sup> Maxwell, Theory of Heat, p. 287.



relation of hygroscopic moisture to the relative surface area presented by different soils, determined by a method which depends directly upon the amount of surface, will be made the subject of a later investigation.

#### PROPERTIES OF FILMS.

Since the movement and adjustment of water among the soil grains depend upon forces whose action is the same as if a uniform tension existed in the water surfaces, it is very important to study the properties of these surfaces and their effectiveness under different conditions. Water is not a convenient substance to use in the experimental study of films, since its high surface tension and low superficial viscosity make the film unstable, except when very small surfaces are used. An ordinary soap solution, on the other hand—or better, a solution of oleate of soda to which a quantity of glycerin has been added—gives films of great stability and highly adapted to experiment.<sup>1</sup>

The most familiar form of a soap film is the spherical surface assumed by a bubble, as representing the least surface area under the given conditions. The fact that the bubble will contract and finally become a plane film across the end of the pipe shows that there is a tension in the film which produces a pressure upon the air inside the bubble. Suppose, now, that a large and a small bubble are connected by means of a short pipe of circular cross section. It might be expected that the large bubble would blow the smaller one out until the two were of equal size. On the contrary, the opposite takes place. The small bubble contracts until it becomes a film across the end of the pipe. This film, however, is not a plane as in the first case, since on one side there is the pressure due to the large bubble and on the other the atmospheric pressure. It will consequently assume a spherical surface identical with a segment of the large bubble, of which it in reality forms a part, separated from it by means of the pipe.

The action of the smaller bubble in blowing out the larger, or the pressure in the smaller bubble being greater than in the larger, might have been to some extent anticipated if we had considered more fully the plane film across the end of the open pipe. Suppose such a film stretched across a circular ring of wire. This film might be looked upon as part of the surface of a great bubble of infinite radius. Since the pressure on both sides of the film is the same, it follows that in such a bubble the pressure inside would equal the pressure outside. It is not necessary, therefore, for the film to exert any pressure in either direction in order to maintain the equilibrium of the system. For a plane surface or a sphere of infinite radius it is evident that the pressure of the film is zero.

Let us now consider a ring of smaller radius than the first, contain-

<sup>1</sup> For particulars in regard to the preparation of the soap solution, see *Soap Bubbles*, C. V. Boys, published by Young & Co., New York. This book also contains an account of many interesting and instructive experiments with films.

ing a segment of a bubble of such radius that the surface area of the segment is just equal to the surface area of the plane film held in the first ring. The surface energy is the same in the two cases. In the first case, however, it is directed in the plane of the ring, while in the latter case it can be divided into two portions, one of which acts in the plane of the ring and the other in a direction at right angles to this plane. This film, therefore, exerts a pressure, while the other does not, although both have the same energy. This pressure, therefore, evidently depends upon the form of the surface. This is of the greatest importance in connection with the problems relative to the movement of soil moisture. It is the form of the capillary surface which determines whether or not it is in equilibrium with other water surfaces in the soil—whether it shall expand or contract, advance or recede.

#### PRESSURE OF A FILM.

It has been shown that the pressure which a film is able to exert is dependent on the form of the film. It is now of interest and importance to determine in what way the pressure is related to the curvature of the film and to its surface tension, in order that the action of films under certain conditions in the soil may be determined.

Suppose a cylindrical film of radius  $r$  and pass a plane through it at right angles to the axis.<sup>1</sup> Suppose the section to be represented by the plane of the paper. (See fig. 2.) Let  $ab$  be a small portion of the section of the film thus formed. Let  $T$  represent the surface tension directed tangentially at  $a$  and  $b$  in the plane of the section, the film considered being taken of unit width perpendicular to this plane and of length  $ab$ .

The resultant of the two components of  $T$  is  $cd$  or  $P$ .

$$\begin{aligned}\text{Then } P &= 2T \sin \frac{1}{2} \theta \\ \sin \frac{1}{2} \theta &= \frac{ab}{2r} \therefore P = T \frac{ab}{r}.\end{aligned}$$

If  $ab=1$ , we have the unit area of surface and  $P = \frac{T}{r}$ ; or the pressure varies directly as the surface tension and the curvature, the latter being the reciprocal of the radius. In general the pressure on any film at any point can be expressed by

$$P = T \left( \frac{1}{r_1} + \frac{1}{r_2} \right)$$

in which  $r_1$  and  $r_2$  represent the radii of curvature of the sections formed by passing two normal planes at right angles through the point.

For a sphere  $r_1=r_2$ , and we have  $P = \frac{2T}{r}$ , or the pressure of a spherical surface is twice the surface tension divided by the radius. It is now easy to understand why the smaller bubble blows the larger one out.

<sup>1</sup> In what follows, but one surface of the film is considered. Where two concentric surfaces exist, as in bubbles, the pressure would be doubled.

Since the surface tension is the same on both bubbles, if the smaller bubble is only one-half the diameter of the larger its internal pressure would be twice as much.

#### SURFACE OF NO PRESSURE.

It is evident from the equation that there are only two surfaces of revolution in which the pressure can be zero. For if we put  $P$  equal to zero it is necessary either that both  $r_1$  and  $r_2$  be equal to infinity, or that  $r_1 = -r_2$  in order that the equation may be satisfied. The first case is satisfied by the plane, which is a sphere of infinite radius. A plane film therefore has no pressure. The other case is satisfied by the catenoid surface in which  $r_1 = -r_2$ , that is, in which the radii are numerically equal and on opposite sides of the film.

This figure is of much importance in the consideration of films. It is generated by the revolution of the catenary curve (*catena*, chain) about its directrix. This curve is that represented by a chain or flexible cord of uniform weight hanging from two horizontal supports at a distance from each other less than the length of the cord. The equation is:

$$y = \frac{a}{2} \left( e^{\frac{x}{a}} - e^{-\frac{x}{a}} \right)$$

in which  $a$  is the distance from the

lowest point of the curve to the directrix, which is taken to coincide with the axis of  $x$ , and  $e$  is the base of the natural system of logarithms. It is evident that the curve is symmetrical about the axis of  $y$  and that its intercept on the axis of  $y$  is equal to the value of  $a$  chosen. This curve is illustrated in fig. 3, in which different forms are obtained by assigning different values to  $a$ . They represent the different positions assumed by a chain under different tensions, or by a film having different surface tensions.

One of the best examples of the catenoid is found in a soap film stretched between two equal circular rings of wire so placed that their planes are parallel and their edges bound a rectilinear cylinder. This can be easily prepared by wetting the rings in the soap solution and blowing a bubble upon them. The film inside each ring is then broken with a hot wire, after which the rings may be separated. Since the

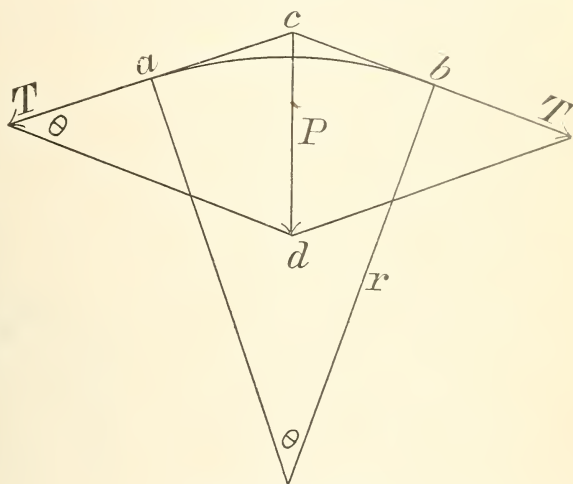


FIG. 2.—Pressure of a film.

pressure is now the same on both sides of the film we have a surface of no pressure. This surface is stable only when the tangents to the catenary at its extremities intersect before reaching the axis of the figure. The accompanying illustration (fig. 4) is taken from a photograph of a film prepared in the manner described.

A section of the catenoid midway between the wire rings and parallel to them is a circle. A circle of this diameter if cut out from a piece of paper and held with its edge against the catenary would exactly fit the surface for a short distance each side of the center, or the surface is one of no curvature. Since the potential energy of any system always tends to reach its minimum, the film tends to contract to its smallest possible

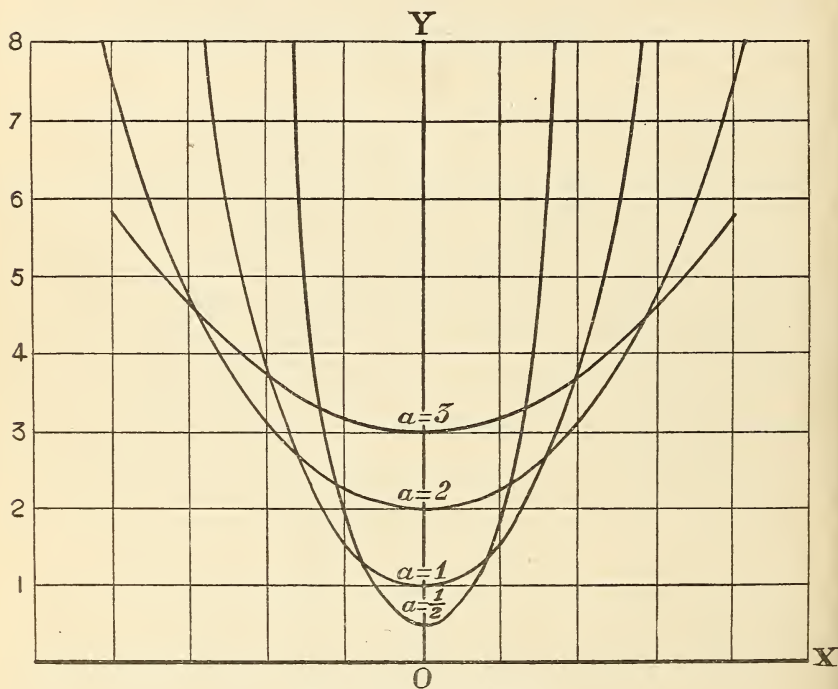


FIG. 3.—Forms of the catenary.

area and the film between the two rings represents the surface of least area which can be formed between them.

Let a small soap bubble be blown between two rings as in the preparation of the catenoid soap film. Now separate the rings until the film between the rings forms a cylinder with straight walls, as in fig. 5. The pressure inside the cylinder is evidently greater than the atmospheric pressure, as is shown by the bulging outward of the films inside the rings. In fact, from a measurement of the curvature of these spherical segments at the end of the cylinder, we find the pressure in the cylinder to be equal to that of a spherical bubble of twice the diameter of the cylinder.



Now insert the pipe in the cylinder through one of the ends and allow the pressure inside and out to become equalized. The films inside the rings become plane, since the plane is a surface of no pressure. The cylinder degenerates into a catenoid, also a surface of no pressure. Suppose now the pressure is reduced still more, so that the films inside the rings bulge inward, showing the pressure on the inside to be less than outside. This gives a surface having a more pronounced waist than the catenoid. It represents the form assumed by the free surface of a liquid between two solid surfaces, which it wets and is therefore called



FIG. 4.—Catenoidal film.

a capillary surface. This form of surface is therefore the one that is met with in the capillary space formed by the contact of two soil grains. The generating curve of a capillary surface is known as the "elastic curve," from its identity with the curve formed by a straight spring of uniform flexibility when its ends are acted upon by equal and opposite forces. Some of the forms assumed by this curve under different conditions can be easily obtained with a steel spring.<sup>1</sup>

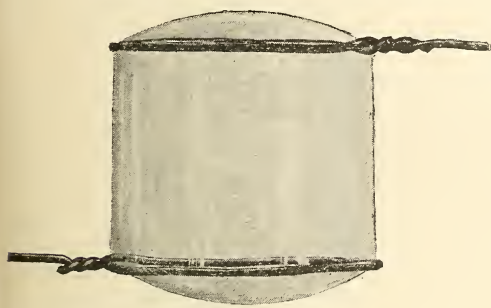


FIG. 5.—Cylindrical film.

To summarize briefly, the pressure which a film may exert depends upon its surface tension and its form: (1) The pressure of the film may be inward, as in the case of the cylinder, indicated by the films inside the rings being forced outward; (2) the pressure may be zero, as in the case of the catenoid, the films inside the rings

being plane; (3) the pressure may be outward, as in the case of surfaces generated by the elastic curve, the films within the ring being drawn inward.

If the interior of two closed films of unequal size, both having a pressure inward and both free to move, are connected, the smaller film having the greater pressure will contract, forcing the larger film to expand. This is illustrated by two soap bubbles or by the coalescence of drops of mercury. If the two films have a pressure outward, the smaller film,

<sup>1</sup>For tracings obtained in this way see Thomson & Tait, *Natural Philosophy*, Part I, p. 148.



having the greater curvature, will expand until the two films become equal. This is what would take place in surfaces generated by the elastic curve—that is, capillary surfaces—if these surfaces were free to move, and is of the greatest importance in the adjustment of the water content of a soil among the capillary spaces.

#### FORM OF WATER SURFACE BETWEEN TWO SOIL GRAINS.

The manner in which water is held in a soil may now be considered. Suppose the soil grains to be momentarily separated so that no two grains are in contact. Each grain carries with it a small amount of moisture, which, through the agency of surface tension, spreads over the surface of the grain. Suppose now that two of these grains, which may be assumed spherical in form, are once more brought in contact. The water is drawn into the capillary space formed between the spheres and forms between the grains a capillary water surface.

The collection of a portion of the water previously distributed over the surface of the grains into this capillary space may be explained from a consideration of the curvature and pressure of the water surfaces, which will hereafter be designated as films. When the two spheres are brought in contact, the films in meeting form a surface of very great curvature having a pressure outward. Since the two films on the surface of the spheres have a pressure inward, the water moves rapidly toward the capillary space. As the new surface increases, its pressure becomes less and the movement of water becomes slower, finally ceasing when the pressure is not sufficient to overcome the resistance offered by the films.

The arrangement of the water between the spheres may also be considered from the standpoint of the potential energy of the system. It has been shown that a film between two rings assumes the form of the catenoid, as representing the least surface, and consequently the minimum potential energy. Therefore, any liquid surface held between two parallel planes will tend, so far as the conditions will admit, to approach the catenoid in form, which is the limiting form of the capillary surface. This is well illustrated by the ink between the blades of a right-line ruling pen. These parallel planes may be supposed to separate gradually as the amount of water increases, and this gives the condition on the spherical surfaces.

On the assumption that no evaporation can take place and that the spheres are entirely covered with moisture before being brought in contact, it follows that the spheres will still be covered after contact, although the film will be much thinner than before. The amount of diminution in the surface, due to the contact of the spheres, is evidently equal to the difference between the combined area of the original water surfaces and the combined area of the spherical segments outside the capillary surface, together with the area of the capillary surface. If the thickness of the original films on the spheres was small in compari-

son with their radii, the diminution in surface is practically equal to the difference between the combined areas of the spherical surfaces inclosed in the capillary space and the area of the inclosing capillary surface. If the equation of the capillary surface, which is too complex to be considered profitably here, is known, the area of the surface can be calculated and the change in potential energy determined.

#### ESTABLISHMENT OF EQUILIBRIUM BETWEEN TWO UNEQUAL MASSES OF CAPILLARY WATER.

Suppose the capillary spaces formed by several spheres in contact to contain different amounts of water. These spheres are supposed originally to have been covered with films and then brought in contact, so that water films will exist on all surfaces which are not submerged. Let one of these films connect the water held in two adjacent capillary spaces containing different quantities of water. This is illustrated

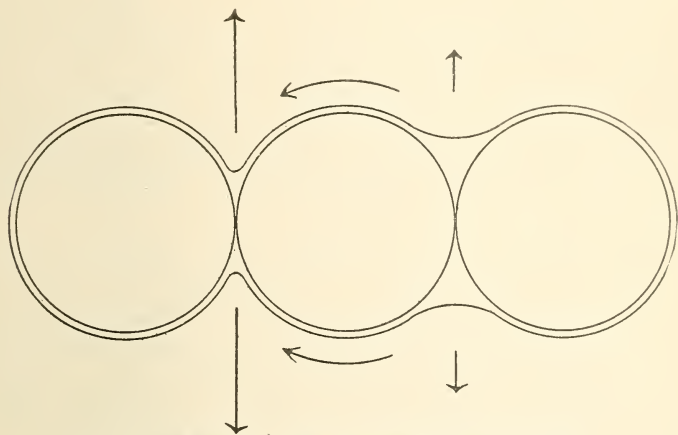


FIG. 6.—Adjustment of water between two capillary spaces.

diagrammatically in fig. 6, which represents a section through the centers of three spheres in contact.

Since the capillary spaces are similar in form, it follows from previous considerations that the surface of the smaller mass of water will have the greater curvature, and consequently the greater pressure outward. The direction and relative magnitude of the pressures of the two films taken within the section are represented by the length of arrows. Since the surface of the lesser mass of water exerts the greater pressure outward, water will move through the connecting film in the direction of the curved arrows from the greater to the lesser mass. This action will continue until the pressure becomes the same, which in this case takes place when the capillary spaces contain equal amounts of water, since the spaces are of the same form. In any case, equilibrium is reached when the two films attain the same curvature.

The rate at which this adjustment of water between two capillary

spaces will take place depends upon the viscosity of the connecting film, the surface tension, and the difference in curvature of the films. The viscosity of the connecting film does not in any way interfere with the final adjustment of the water, but it retards to a greater or less degree the establishment of equilibrium. An increase in either the curvature or the surface tension causes an increase in pressure, as has been previously pointed out.

It is evident that this movement can be extended to any number of capillary spaces through any number of films, so that adjustment takes place over a large mass of soil when disturbing influences are introduced. This change takes place more or less slowly, according to the amount of water present in the soil. If the soil is nearly saturated, so that the films connecting the capillary spaces are short and thick and the capillary spaces themselves are not active, but little resistance is offered to the movement of water and the addition of water at the surface is quickly felt farther down. If, on the other hand, the soil contains but little water, the same amount of water added to the surface, while producing marked changes in the upper layers, will not be felt so quickly at the lower depths on account of the activity of the upper capillary spaces and the length and small cross section of the connecting films. But an adjustment of the water between the upper and lower capillary spaces takes place in this case also until equilibrium is gradually reached.

#### SALTS AS AFFECTING THE MOVEMENT OF WATER IN SOILS.

We have seen the importance of surface tension in opposing the gravitation of capillary water of soils. Any change in the surface tension of the soil moisture tends to bring about an adjustment of the water throughout the whole mass of soil. If the surface tension of the water in the upper layers of a soil is increased, water is drawn toward that point. Since the surface tension of most salt solutions is higher than that of water, and the surface tension increases with the concentration of the solution, it might be expected that any salt used as fertilizer a solution of which has a high surface tension would increase the amount of water in the soil.

It must be remembered, however, that the surface tension of solutions is very greatly decreased by the addition of very small quantities of certain organic substances produced through the decomposition of vegetable matter. This action is especially marked where there are present substances of an oily nature which do not go into solution, but spread out over the surface in an extremely thin film. Owing to such substances being continually produced by the decay of organic matter, the surface tension of the soil moisture is kept very low and could be only slightly influenced by the addition of salts. The application of substances to the soil for the purpose of changing its water content through a change in the surface tension, would not therefore necessarily be productive of marked results.

## TEMPERATURE AS AFFECTING THE MOVEMENT OF WATER IN SOILS.

It was pointed out in a preceding section that the surface tension of water decreases with increase of temperature. Therefore if the bottom of a column of soil in which the water has attained a condition of equilibrium should be cooled, the surface tension of the lower strata would be raised and the water would be drawn toward the bottom, or if the lower strata should be heated the water would tend to move toward the top. The first method of procedure should give the most marked results, since the movement in this case is assisted by gravitation. A movement should also be secured by raising or lowering the temperature of the whole mass of soil uniformly. In the first case the water content of the upper strata would be decreased and in the second case increased. These conclusions are indirectly verified by some interesting experiments of Professor King<sup>1</sup> in experimenting with the fluctuations of ground water in a large, cylindrical, galvanized iron tank. He found that the water in a circular well in the middle of the cylinder rose daily and fell again during the night. The application of cold water to the outside of the cylinder by means of a hose also caused the water in the well to fall. These results are fully consistent with the phenomena of surface tension. When the temperature of the soil was raised the surface tension of the water was lowered and more water was drawn into the lower part of the cylinder, which raised the level of the water in the well. When cold water was applied to the outer surface of the cylinder, the water in the soil was drawn up again through increased surface tension and the level of the water in the well was lowered.

The influence of temperature on the rate of flow of water in saturated soils is very great. This is due to a change in the viscosity of water with temperature, as has been pointed out in the section on viscosity. This property is not only of interest in considering saturated soils, but it is also an important factor in determining the rate of adjustment of water in soils in which saturation is not complete.

## INFLUENCE OF TEXTURE AND STRUCTURE OF SOILS ON THE ACQUIREMENT AND RETENTION OF SOIL MOISTURE.

The limit of the capacity of any soil for water is reached when the surface tension holding the water in the capillary spaces is no longer able to overcome the force of gravity acting on the mass. The relative water capacity of two soils, therefore, depends principally upon the number and size of the capillary spaces. By a capillary space as used here is meant not any interstitial space in the soil structure, but only that portion of it which is near the point of contact of two soil grains. It is that portion in which the bounding walls are close together, separated only by distances of capillary magnitude and consequently most efficient in retaining water. It is evident that in a soil of fine texture

<sup>1</sup> U. S. Department of Agriculture Bul. No. 5, Weather Bureau, pp. 59-61.



the grains might be so close together as to make all the interstitial space capillary in its nature.

The one important factor which determines the acquirement and retention of soil moisture is the curvature of the capillary water surfaces. If equal volumes of two soils are placed in contact, and the curvature of the surface is less in the first than in the second, then water will move from the first to the second, increasing the curvature in one and decreasing it in the other until it becomes the same in both soils. If the second soil contains a greater number of capillary spaces than the first, it will contain more water when equilibrium is established. During the adjustment water will have actually moved from a soil containing a low percentage of water to one having a higher percentage. In no case, however, will water leave a capillary space having a water surface of large curvature to go to a space with a surface of less curvature. It is the form of the surface which determines the movement of the water.

In a form of structure presented by Dr. Soyka, to which reference has previously been made, and in which the spheres are arranged for the greatest amount of interstitial space, there are only about one-half as many points of contact between the grains as in another form of structure given, although the amount of interstitial space in the first case is twice as great. Consequently the second form—the compact soil—would have twice the water capacity of the first, since the number of capillary spaces formed is twice as great. The difference in this case would be due entirely to the structure of the soil, since the texture remains uniform.

In the same manner a soil of fine texture contains many more capillary spaces than a soil which is coarse, and consequently has a much greater water-holding power. In a coarse sandy soil the interstitial spaces are large, allowing percolation and drainage to take place rapidly, but permitting the formation of comparatively few capillary spaces for the storing of water. As the texture becomes finer the interstitial space becomes smaller and the capillary spaces increase in number, and embrace a large proportion of the whole interstitial space. The water capacity of the soil increases and percolation is greatly decreased. The limit is reached when the texture becomes so fine and the structure so close that all the interstitial space becomes capillary in its nature. The capacity of the soil for moisture in this case is reached only when all the interstitial space becomes filled with water, a condition found in some clay soils.

#### DISPLACEMENT OF CAPILLARY WATER THROUGH GRAVITATION.

In considering the adjustment of water among the capillary spaces and the arrangement of the film so as to present as little free space as possible, it was assumed that the action of gravity could be neglected. This is undoubtedly the case in the smaller capillary spaces, but in those existing between the larger soil grains gravitation causes a displacement of the capillary water, so that it is no longer symmetrically



arranged about the point of contact. The action of gravity on the capillary water between two spheres is illustrated in fig. 7.

The model consists of rubber balls about 1 inch in diameter fastened together by small steel pins inserted normal to the surface at the points of contact. The whole is then immersed in cylinder oil and allowed to drain while in a vertical position. While the great size of the balls and the low surface tension of the liquid are conditions not found in soils, it nevertheless illustrates the principle involved. If the liquid had no weight it would be uniformly distributed about the line of centers of spheres. Gravitation, however, causes a distortion of the liquid from a position of symmetry about the point of contact. When the line of centers is horizontal, the liquid moves down until the pressure exerted by the upper portion of the film is equal to that exerted by the lower portion plus the weight of the liquid. The curvature of the upper part of the film must therefore be greater than that of the lower. The upper film is consequently drawn down into the capillary space until it acquires sufficient curvature to support the weight of the liquid and the tension of the opposing film. Reference to the figure will show the upper part of the film far down in the capillary space, while the lower part is well down on the surface of the spheres.

If the line of centers is vertical the curvature from the upper contact will gradually increase downward in order to compensate the effect of the weight of the liquid. This can be seen in the vertical films in the figure, and is well illustrated by the position assumed by a drop of ink between the opened blades of a ruling pen, held so that the blades lie in horizontal planes one above the other.

A consideration of the displacement of the water in the capillary spaces through the influence of gravity serves to give an idea of what takes place when the soil contains more water than the capillary spaces are able to retain. The upper part of the film is then unable to reach a position where it can balance the opposing forces, and water is moved through gravitation until the film is enabled to secure a position in which it can establish equilibrium. Such displacements of capillary water would occur only in the larger capillary spaces where a considerable mass of water would be subjected to the action of gravitation.

In considering the gravitation of water in soils, mention was made (p. 7) of the marked difference in water content of a vertical column of soil at different points along the column. Having developed the principles which regulate the movement of water in soils, the explanation of this non-uniform distribution of water in a vertical column can now be given.



FIG. 7.—Displacement of capillary water through gravitation.

Consider a single vertical column of soil grains as arranged in fig. 7, and suppose the capillary spaces to contain equal amounts of water. The amount of water in each capillary space is assumed to be less than the amount required to saturate it considered by itself, so that if the column were in a horizontal position the water would all be retained. Each capillary surface would in this case be supporting the water in the capillary space, together with the weight of the water in the connecting film on the surface of a soil grain. When the column is raised to a vertical position, the weight of the whole conducting film from top to bottom is thrown for an instant upon the capillary surface nearest the top. This capillary space immediately loses water until the pressure of the surface is equal to the pressure of the surface next below, plus a pressure sufficient to balance the weight of the water in the connecting film between the two surfaces. Both spaces then lose water together, maintaining this difference in pressure until the pressure of the second surface is equal to that of the third plus the pressure necessary to balance the weight of the connecting film, as before. The pressure of the first surface would now be equal to the pressure of the third, plus the pressure necessary to balance the weight of the two connecting films. The action would continue in this manner through each capillary surface until equilibrium is established.

The relative part taken by the different capillary surfaces in supporting the weight of the connecting column can be illustrated by means of the following mechanical analogue. Suppose a series of very thin elastic membranes stretched over circular hoops and supported horizontally one above the other, the distance between the membranes being very small. Now, let a heavy ball be placed upon the upper membrane. This membrane immediately stretches until it reaches the second, then the two stretch together until they touch the third, and so on, until the ball comes to rest. Suppose the membranes all had the same tension before the ball was put in place. After equilibrium is established the upper membrane will have the greatest tension, the second one the next greatest, and so on, analogous to the pressure of the capillary surfaces.

The pressure of the upper capillary surface will always exceed that of the lower surface by the pressure necessary to support the weight of the conducting column. This would necessitate the upper films having a much greater curvature, so that less water could be held in the capillary spaces. The water content should therefore increase uniformly from top to bottom. This has been shown to be the case with coarse sands.

In a soil of fine texture the number of capillary spaces is greatly increased. The pressure exerted by the capillary surfaces would therefore be much greater for the same water content. Consequently, the effect of the weight of the connecting films would be much lessened, and the water content of the soil would be much more uniformly distributed.





